lecture 3 - 2nd law/notion of entropy

- spontaneous macroscopic processes - one way (irreversible)

- Kelvin statement of 2nd law

- statistical mechanics postulate -
  \[ S = k_B \ln \Gamma \] - Boltzmann (microcanonical) entropy

- fundamental hypothesis of statistical mechanics
  - all states same probability -

- toy spin model, evaluation of entropy
  Gaussian distribution of probabilities
  \[ P_M \sim e^{-\frac{1}{2} \frac{M^2}{N}}, \quad P_m \sim e^{-\frac{N m^2}{2}}, \quad m = M/N \]

- example of central limit theorem -
  analogy to random walk
  system most of time at max entropy

- properties of entropy
  
  \[ T \equiv \frac{\partial U}{\partial S} \bigg|_{P,V} \]
  \[ P \equiv \frac{\partial U}{\partial V} \bigg|_{S,N} \]
  \[ S(U,V) \] is equation of state

- thermodynamic identity
  \[ dU = \frac{\partial U}{\partial S} \big|_V dS + \frac{\partial U}{\partial V} \big|_S dV = TdS - PdV \]
  valid for any process, reversible or not

- for reversible process \( dS = \frac{\delta Q}{T}, \quad \delta W = -PdV \)

- thermodynamical definition of entropy, for reversible path,
  \[ S_2 - S_1 = \int_{(1)}^{(2)} \frac{\delta Q}{T} \]
• entropy of ideal gas - heuristic derivation

\[ S = k_B \ln \Gamma, \quad \Gamma = cV^N U^{3N/2} \]  

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• toy spin model, \( S(U, M) \rightarrow M = N\mu \tanh(\mu B/k_B T) \)
  - microcanonical ensemble - fixed \( U, V, N \)

• 3rd law - Nernst - \( T \rightarrow 0, \quad S \rightarrow 0 \)