- ideal gas equation of state from kinetic theory

$$PV = Nk_BT \quad (1)$$

$$\frac{1}{2}m\overline{v^2} \equiv \frac{3}{2}k_BT \quad (2)$$

- a theorem (that relates susceptibilities)
  $f(x, y, z) = 0$

$$\frac{\partial x}{\partial y}|_z \frac{\partial y}{\partial z}|_x = -\frac{\partial x}{\partial z}|_y \quad (3)$$

- 1st law - conservation of energy
  $dU = \delta W + \delta Q$

- reversible work $W_{AB} = -\int_A^B PdV$

- specific heat -

$$C_V = \frac{1}{M}\frac{\delta Q}{dT}|_V = \frac{1}{M}\frac{\partial U}{\partial T}|_V \quad (4)$$

$$C_P = \frac{1}{M}\frac{\delta Q}{dT}|_P = \frac{1}{M}[\frac{\partial U}{\partial T}|_P + P\frac{\partial V}{\partial T}|_P] \quad (5)$$

- ideal gas - $U(T)$
  $U = \frac{3}{2}Nk_BT$ - monoatomic
  $U = \frac{5}{2}Nk_BT$ - diatomic

- $S = k_B \ln \Gamma$ - Boltzmann (microcanonical) entropy

- $T \equiv \frac{\partial U}{\partial S}|_{P,V}$
  $P \equiv \frac{\partial U}{\partial V}|_{S,N}$

- thermodynamic identity

$$dU = \frac{\partial U}{\partial S}|_V dS + \frac{\partial U}{\partial V}|_S dV = TdS - PdV \quad (6)$$

- for reversible process $dS = \frac{\delta Q}{T}, \quad \delta W = -PdV$
- in an irreversible process

$$\Delta S > \frac{\delta Q}{T} \tag{7}$$

- in a cyclic transformation $\Delta S = 0$
  irreversible

$$\sum_i \frac{Q_i}{T_i} < 0 \tag{8}$$

  reversible

$$\sum_i \frac{Q_i}{T_i} = 0 \tag{9}$$

- cyclic Carnot engine - efficiency η

$$\eta = \frac{|W|}{|Q_h|}, \eta \leq 1 - \frac{T_c}{T_h} \tag{10}$$

- for reversible engine $\eta = 1 - \frac{T_c}{T_h}$
- cyclic Carnot refrigerator - efficiency η'

$$\eta' = \frac{|Q_c|}{|W|}, \eta' \leq \frac{T_c}{T_h - T_c} \tag{11}$$

- Thermodynamic Functions $\rightarrow$ Maxwell relations
- internal energy $U(S,V), dU = TdS - PdV$
- "Helmholtz" free energy $F(T,V) = U - TS, dF = -SdT - PdV$
- enthalpy $H(P,S) = U + PV, dH = TdS + VdP$
- "Gibbs" free energy $G(P,T) = U + PV - TS, dG = VdP - SdT$
- Clausius-Clapeyron relation $\frac{dP}{dT} = \frac{L}{T\Delta V}$
- Canonical Ensemble
  probability to find system at state n of energy ϵ_n

$$p_n = \frac{e^{-\frac{\epsilon_n}{k_B T}}}{Z} \tag{12}$$

- partition function $Z = \sum_n e^{-\frac{\epsilon_n}{k_B T}}$

$$\frac{p_n}{p_m} = \frac{e^{-\frac{\epsilon_n}{k_B T}}}{e^{-\frac{\epsilon_m}{k_B T}}} = e^{-\frac{\epsilon_n - \epsilon_m}{k_B T}} \tag{13}$$

$$U = \sum_n p_n \epsilon_n \tag{14}$$

$$S = -k_B \sum_n p_n \ln p_n \tag{15}$$

- free energy $F = U - TS = -k_B T \ln Z$
- free energy (and thus Z) generates all thermodynamic quantities

$$S = -\frac{\partial F}{\partial T}|_V \tag{16}$$

$$U = -T^2 \frac{\partial}{\partial T}(\frac{F}{T}) \tag{17}$$

$$C_V = -T^2 \frac{\partial^2 F}{\partial T^2}|_V \tag{18}$$

- Boltzmann distribution- (p,q)

$$dw = \frac{e^{-\epsilon(p,q)/k_B T}}{Z} dpdq$$

$$Z = \frac{1}{h} \int dpdq e^{-\epsilon(p,q)/k_B T}$$

$$\bar{f} = \frac{\int dpdq f(p,q) e^{-\beta\epsilon(p,q)}}{\int dpdq e^{-\beta\epsilon(p,q)}}$$

- Grand Canonical Ensemble
  probability to find system at state n_N of N particles and energy E_{n_N}

$$p_{n_N} = A e^{-\beta(E_{n_N} - \mu N)} \tag{19}$$

- Landau potential

$$\Omega = -k_B T \ln \sum_N \sum_{n_N} e^{-\beta(E_{n_N} - \mu N)} \tag{20}$$

- independent quantum particles
  Bose-Einstein statistics

$$n_\nu = \frac{1}{e^{\beta(\epsilon_\nu - \mu)} - 1} \tag{21}$$

  Fermi-Dirac statistics

$$n_\nu = \frac{1}{1 + e^{\beta(\epsilon_\nu - \mu)}} \tag{22}$$

- translationally invariant system

$$\sum_k \to \frac{L}{2\pi} \int dk \tag{23}$$