lecture 7 - applications

paramagnet

- $\epsilon = -\vec{\mu}\vec{B}$

$$\begin{aligned}
\epsilon_+ &= = -\mu B, \quad \mu_+ = +\mu, \quad \epsilon_+ = e^{+\mu B/k_B T}/Z \\
\epsilon_- &= = +\mu B, \quad \mu_- = -\mu, \quad \epsilon_- = e^{-\mu B/k_B T}/Z \\
Z &= e^{+\mu B/k_B T} + e^{-\mu B/k_B T} = 2\cosh(\mu B/k_B T) \\
\bar{\mu} &= p_+\mu_+ + p_-\mu_- = \mu \tanh(\mu B/k_B T) \\
M &= N\bar{\mu} = N\mu \tanh(\mu B/k_B T) \\
&\to N\mu(\frac{\mu B}{k_B T}), \quad \frac{\mu B}{k_B T} \to 0
\end{aligned}$$

- magnetic susceptibility - Curie law

$$\begin{aligned}
\chi &= \frac{\partial M}{\partial B}|_T = N\frac{\mu^2}{k_B T}\frac{1}{\cosh^2(\mu B/k_B T)} \\
\mu B/k_B T \quad << 1 &\to \chi \sim N\frac{\mu^2}{k_B T}, 1/\chi \sim T(Curie) \\
\mu B/k_B T \quad >> 1 &\to \chi \sim N\frac{\mu^2}{k_B T}4e^{-2\mu B/k_B T}
\end{aligned}$$

2-level system/finite spectrum

- $N, \quad \epsilon_1, \quad \epsilon_2$

$$\begin{aligned}
Z &= Z_1^N, \quad Z_1 = e^{-\beta\epsilon_1} + e^{-\beta\epsilon_2}, \quad \beta = 1/k_B T \\
F &= -k_B T \ln Z = -k_b N T \ln(Z_1) = -k_B N T \ln(e^{-\beta\epsilon_1} + e^{-\beta\epsilon_2}) \\
U &= -\frac{\partial \ln Z}{\partial \beta} = N\frac{\epsilon_1 e^{-\beta\epsilon_1} + \epsilon_2 e^{-\beta\epsilon_2}}{e^{-\beta\epsilon_1} + e^{-\beta\epsilon_2}} \\
S/N &= \frac{U - F}{T} = \frac{1}{T}\frac{\epsilon_1 e^{-\beta\epsilon_1} + \epsilon_2 e^{-\beta\epsilon_2}}{e^{-\beta\epsilon_1} + e^{-\beta\epsilon_2}} + \ln(e^{-\beta\epsilon_1} + e^{-\beta\epsilon_2})
\end{aligned}$$

- notion of negative temperature

$$\begin{aligned}
&T \to 0^+, \quad \beta \to +\infty, \quad U/N \to \epsilon_1, \; S/N \to 0 \\
&T \to +\infty, \quad \beta \to 0^+, \quad U/N \to \frac{\epsilon_1+\epsilon_2}{2}, \; S/N \to \ln 2 \\
&T \to -\infty, \quad \beta \to 0^-, \quad U/N \to \frac{\epsilon_1+\epsilon_2}{2}, \; S/N \to \ln 2 \\
&T \to 0^-, \quad \beta \to -\infty, \quad U/N \to \epsilon_2, \; S/N \to 0
\end{aligned}$$

- "Schottky" specific heat

$$\begin{aligned}
& & \epsilon_1 = -\epsilon \quad \epsilon_2 = +\epsilon \\
u &=& U/N = -\epsilon \tanh(\beta\epsilon) \\
c &=& \partial u/\partial T = k_B(\frac{\epsilon}{k_B T})^2 \frac{1}{\cosh^2(\epsilon/k_B T)} \\
& & T \to 0, c \sim k_B(\frac{\Delta}{k_B T})^2 e^{-\Delta/k_B T}, \quad \Delta = 2\epsilon \\
& & T \to \infty, c \sim k_B(\frac{\Delta}{k_B T})^2
\end{aligned}$$

N-quantum oscillators/Einstein model

- $\epsilon_n = (n+1/2)\hbar\omega, \quad n = 0, 1, 2, ..., +\infty$

$$\begin{aligned}
Z &=& (\sum_{n=0}^{+\infty} e^{-\beta\epsilon_n})^N = Z_1^N \\
Z_1 &=& \frac{1}{2\sinh(\beta\hbar\omega/2)} \\
F &=& -k_B T \ln Z = k_B T N \ln(2\sinh(\beta\hbar\omega/2)) \\
U &=& N\frac{\hbar\omega}{2}\coth(\frac{\beta\hbar\omega}{2}) \\
c &=& k_B(\frac{\hbar\omega}{2k_B T})^2 \frac{1}{\sinh^2(\hbar\omega/2k_B T)} \\
\hbar\omega &<<& k_B T \to c \sim k_B \\
\hbar\omega &>>& k_B T \to c \sim k_B(\frac{\hbar\omega}{2k_B T})^2 e^{-\hbar\omega/k_B T}
\end{aligned}$$