lecture 4 - Carnot cycle

- in an irreversible process

$$\Delta S > \frac{\delta Q}{T} \tag{1}$$

- 2nd law - Clausius
  - in a cyclic transformation $\Delta S = 0$

$$\sum_i \frac{Q_i}{T_i} = \oint \frac{\delta Q}{T} \leq 0 \tag{2}$$

- cyclic Carnot engine - efficiency η

$$\begin{aligned}
dU &= \delta Q + \delta W && (3)\\
0 &= Q_h + Q_c + W && (4)\\
0 &= |Q_h| - |Q_c| - |W| && (5)\\
\eta = \frac{|W|}{|Q_h|} &= \frac{|Q_h| - |Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|} && (6)\\
\frac{Q_h}{T_h} + \frac{Q_c}{T_c} &= \frac{|Q_h|}{T_h} - \frac{|Q_c|}{T_c} \leq 0 && (7)\\
\eta &\leq 1 - \frac{T_c}{T_h} && (8)
\end{aligned}$$

- Carnot theorem - for reversible engine
  $\eta = 1 - \frac{T_c}{T_h}$

- cyclic Carnot refrigerator - efficiency η'

$$\begin{aligned}
dU &= = \delta Q + \delta W && (9)\\
0 &= = Q_h + Q_c + W && (10)\\
0 &= -|Q_h| + |Q_c| + |W| && (11)\\
\eta' &= \frac{|Q_c|}{|W|} = \frac{|Q_c|}{|Q_h| - |Q_c|} && (12)\\
\frac{1}{\eta'} &= \frac{|Q_h|}{|Q_c|} - 1 && (13)
\end{aligned}$$

$$\frac{Q_h}{T_h} + \frac{Q_c}{T_c} = -\frac{|Q_h|}{T_h} + \frac{|Q_c|}{T_c} \leq 0 \tag{14}$$

$$\eta' \leq \frac{T_c}{T_h - T_c} \tag{15}$$

- reversible Carnot cycle
  (1) isothermal expansion $Q_1 = Nk_B \ln(\frac{V_b}{V_a})$
  (2) adiabatic expansion $T_1 V_b^{\gamma-1} = T_2 V_c^{\gamma-1}$
  (3) isothermal compression $Q_2 = -Nk_B \ln(\frac{V_c}{V_d})$
  (4) adiabatic compression $T_2 V_d^{\gamma-1} = T_1 V_a^{\gamma-1}$

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0 \tag{16}$$

- entropy point of view